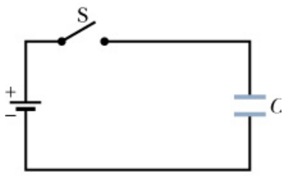
Mr. Shapiro

AP Physics, Spring 2018
Capacitance Solution Set, Cha. 25
#1,4,5,8,12,18,26,27,30,38,45,48
due Mon. 2/23



1. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery. Thus,

$$q = (25 \times 10^{-6}\text{ F})(120\text{ V}) = 3.0 \times 10^{-3}\text{ C}.$$

4. We use $C = A\varepsilon_0/d$.

(a) Thus,

$$d = \frac{A\varepsilon_0}{C} = \frac{\left(1.00\,\text{m}^2\right)\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}{1.00\,\text{F}} = 8.85\times10^{-12}\text{ m}.$$

(b) Since d is much less than the size of an atom ($\sim 10^{-10}$ m), this capacitor cannot be constructed.

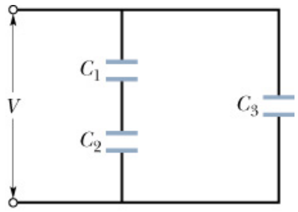
5. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 \quad \Rightarrow \quad R' = 2^{1/3}R.$$

The new capacitance is

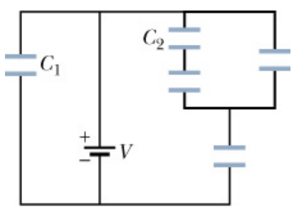
$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3} R = 5.04\pi\varepsilon_0 R.$$

With $R = 2.00$ mm, we obtain $C = 5.04\pi\left(8.85\times10^{-12}\text{ F/m}\right)\left(2.00\times10^{-3}\text{ m}\right) = 2.80\times10^{-13}\text{ F}$.



8. The equivalent capacitance is

$$C_{\text{eq}} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 4.00\mu\text{F} + \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 7.33\,\mu\text{F}.$$



12. (a) The potential difference across C_1 is $V_1 = 10.0$ V. Thus,

$$q_1 = C_1 V_1 = (10.0\ \mu\text{F})(10.0\ \text{V}) = 1.00 \times 10^{-4}\ \text{C}.$$

(b) Let $C = 10.0\ \mu$F. We first consider the three-capacitor combination consisting of C_2 and its two closest neighbors, each of capacitance C. The equivalent capacitance of this combination is

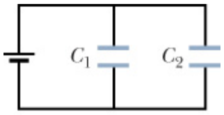
$$C_{\text{eq}} = C + \frac{C_2 C}{C + C_2} = 1.50\ C.$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C + C_{eq}} = \frac{CV_1}{C + 1.50\ C} = 0.40 V_1.$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage difference across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2 V_2 = (10.0\mu\text{F})\left(\frac{10.0\,\text{V}}{5}\right) = 2.00 \times 10^{-5}\ \text{C}.$$



18. Eq. 23-14 applies to each of these capacitors. Bearing in mind that $\sigma = q/A$, we find the total charge to be

$$q_{\text{total}} = q_1 + q_2 = \sigma_1 A_1 + \sigma_2 A_2 = \varepsilon_0 E_1 A_1 + \varepsilon_0 E_2 A_2 = 3.6 \text{ pC}$$

where we have been careful to convert cm^2 to m^2 by dividing by 10^4.

26. (a) The capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(40\times10^{-4}\text{ m}^2\right)}{1.0\times10^{-3}\text{ m}} = 3.5\times10^{-11}\text{ F} = 35\,\text{pF}.$$

(b) $q = CV = (35 \text{ pF})(600 \text{ V}) = 2.1 \times 10^{-8} \text{ C} = 21 \text{ nC}$.

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(35\,\text{pF})(21\,\text{nC})^2 = 6.3\times10^{-6}\,\text{J} = 6.3\,\mu\text{J}$.

(d) $E = V/d = 600 \text{ V}/1.0 \times 10^{-3} \text{ m} = 6.0 \times 10^5 \text{ V/m}$.

(e) The energy density (energy per unit volume) is

$$u = \frac{U}{Ad} = \frac{6.3\times10^{-6}\,\text{J}}{\left(40\times10^{-4}\text{m}^2\right)\left(1.0\times10^{-3}\text{m}\right)} = 1.6\ \text{J/m}^3.$$

27. The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel, the potential difference V across the capacitors is the same and the total energy is

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\text{ F} + 4.0\times10^{-6}\text{ F}\right)(300\text{ V})^2 = 0.27\,\text{J}.$$

30. (a) The charge q_3 in the Figure is $q_3 = C_3V = (4.00\,\mu\text{F})(100\text{ V}) = 4.00\times10^{-4}\text{ C}$.

(b) $V_3 = V = 100$ V.

(c) Using $U_i = \frac{1}{2}C_iV_i^2$, we have $U_3 = \frac{1}{2}C_3V_3^2 = 2.00\times10^{-2}\text{ J}$.

(d) From the Figure,

$$q_1 = q_2 = \frac{C_1C_2V}{C_1+C_2} = \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})(100\text{ V})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}} = 3.33\times10^{-4}\text{C}.$$

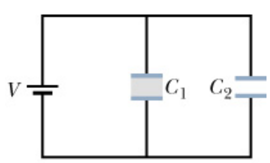
(e) $V_1 = q_1/C_1 = 3.33 \times 10^{-4}$ C/10.0 μF = 33.3 V.

(f) $U_1 = \frac{1}{2}C_1V_1^2 = 5.55\times10^{-3}\text{ J}$.

(g) From part (d), we have $q_2 = q_1 = 3.33\times10^{-4}\text{C}$.

(h) $V_2 = V - V_1 = 100\text{ V} - 33.3\text{ V} = 66.7$ V.

(i) $U_2 = \frac{1}{2}C_2V_2^2 = 1.11\times10^{-2}\text{ J}$.



38. Each capacitor has 12.0 V across it, so Eq. 25-1 yields the charge values once we know C_1 and C_2. From Eq. 25-9,

$$C_2 = \frac{\varepsilon_0 A}{d} = 2.21\times10^{-11}\text{ F},$$

and from Eq. 25-27,

$$C_1 = \frac{\kappa\varepsilon_0 A}{d} = 6.64\times10^{-11}\text{ F}.$$

This leads to $q_1 = C_1V_1 = 8.00\times10^{-10}$ C and $q_2 = C_2V_2 = 2.66\times10^{-10}$ C. The addition of these gives the desired result: $q_{tot} = 1.06\times10^{-9}$ C. Alternatively, the circuit could be reduced to find the q_{tot}.

45. (a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\varepsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\varepsilon_0 A/C$ and

$$E = \frac{VC}{\kappa\varepsilon_0 A} = \frac{(50\ \text{V})(100\times10^{-12}\ \text{F})}{5.4(8.85\times10^{-12}\ \text{F/m})(100\times10^{-4}\ \text{m}^2)} = 1.0\times10^4\ \text{V/m}.$$

(b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12}\ \text{F})(50\ \text{V}) = 5.0 \times 10^{-9}\ \text{C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A},$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \varepsilon_0 AE = 5.0\times10^{-9}\,\text{C} - (8.85\times10^{-12}\ \text{F/m})(100\times10^{-4}\,\text{m}^2)(1.0\times10^4\ \text{V/m}) \\ &= 4.1\times10^{-9}\,\text{C} = 4.1\,\text{nC}. \end{aligned}$$

48. (a) We apply Gauss's law with dielectric: $q/\varepsilon_0 = \kappa EA$, and solve for κ:

$$\kappa = \frac{q}{\varepsilon_0 EA} = \frac{8.9\times10^{-7}\ \text{C}}{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(1.4\times10^{-6}\ \text{V/m})(100\times10^{-4}\,\text{m}^2)} = 7.2.$$

(b) The charge induced is

$$q' = q\left(1-\frac{1}{\kappa}\right) = (8.9\times10^{-7}\ \text{C})\left(1-\frac{1}{7.2}\right) = 7.7\times10^{-7}\ \text{C}.$$