Mr. Shapiro

AP Physics, Spring 2018
Magnetic Field Solution Set, Cha. 28 # 3,10,24,35,45,82
due Mon. 3/16

3. (a) The force on the electron is

$$\vec{F}_B = q\vec{v}\times\vec{B} = q\left(v_x\hat{\mathrm{i}}+v_y\hat{\mathrm{j}}\right)\times\left(B_x\hat{\mathrm{i}}+B_y\vec{j}\right) = q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}}$$
$$=\left(-1.6\times10^{-19}\,\mathrm{C}\right)\left[\left(2.0\times10^{6}\,\mathrm{m/s}\right)\left(-0.15\,\mathrm{T}\right)-\left(3.0\times10^{6}\,\mathrm{m/s}\right)\left(0.030\,\mathrm{T}\right)\right]$$
$$=\left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}.$$

Thus, the magnitude of $\vec{F}_B$ is 6.2×10^{14} N, and $\vec{F}_B$ points in the positive z direction.

(b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\vec{F}_B$ has the same magnitude but points in the negative z direction, namely, $\vec{F}_B = -\left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}.$

10. (a) The force due to the electric field ($\vec{F}= q\,\vec{E}$) is distinguished from that associated with the magnetic field ($\vec{F}= q\,\vec{\mathrm{v}}\times\vec{B}$) in that the latter vanishes at the speed is zero and the former is independent of speed. The graph (Fig.28-34) shows that the force (y-component) is negative at v = 0 (specifically, its value is -2.0×10^{-19} N there) which (because $q = -e$) implies that the electric field points in the $+y$ direction. Its magnitude is

$$E = (2.0 \times 10^{-19})/(1.60 \times 10^{-19}) = 1.25 \text{ V/m}.$$

(b) We are told that the x and z components of the force remain zero throughout the motion, implying that the electron continues to move along the x axis, even though magnetic forces generally cause the paths of charged particles to curve (Fig. 28-11). The exception to this is discussed in section 28-3, where the forces due to the electric and magnetic fields cancel. This implies (Eq. 28-7) $B = E/v = 2.50 \times 10^{-2}$ T.

For $\vec{F} = q\,\vec{\mathrm{v}}\times\vec{B}$ to be in the opposite direction of $\vec{F} = q\,\vec{E}$ we must have $\vec{\mathrm{v}}\times\vec{B}$ in the <u>opposite</u> direction from $\vec{E}$ which points in the $+y$ direction, as discussed in part (a). Since the velocity is in the $+x$ direction, then (using the right-hand rule) we conclude that the magnetic field must point in the $+z$ direction ($\hat{\mathrm{i}}\times\hat{\mathrm{k}} = -\hat{\mathrm{j}}$). In unit-vector notation, we have $\vec{B}=(2.50\times10^{-2}\ \mathrm{T})\hat{\mathrm{k}}$.

24. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v} \times \vec{B}$ points leftward, which indeed seems to be the direction it is "pushed''; therefore, $q > 0$ (it is a proton).

(a) Eq. 28-17 becomes $T = 2\pi m_{\mathrm{p}} / e|\vec{B}|$, or

$$2\left(130\times10^{-9}\right)=\frac{2\pi\left(1.67\times10^{-27}\right)}{\left(1.60\times10^{-19}\right)|\vec{B}|}$$

which yields $\left|\vec{B}\right| = 0.252\,\mathrm{T}$.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$. Since the period T does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$). Thus, $t = T/2 = 130$ ns, again.

35. (a) The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. Thus,

$$iLB = mg \Rightarrow i = \frac{mg}{LB} = \frac{(0.0130\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)}{(0.620\,\mathrm{m})(0.440\,\mathrm{T})} = 0.467\,\mathrm{A}.$$

(b) Applying the right-hand rule reveals that the current must be from left to right.

45. We use Eq. 28-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline F_N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. We take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

$$mg\sin\theta - f = ma.$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B\sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B\sin\theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate f from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = 2NirL$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\text{kg})(9.8\,\text{m/s}^2)}{2(10.0)(0.100\,\text{m})(0.500\,\text{T})} = 2.45\,\text{A}.$$

82. (a) For the magnetic field to have an effect on the moving electrons, we need a non-negligible component of $\vec{B}$ to be perpendicular to $\vec{v}$ (the electron velocity). It is most efficient, therefore, to orient the magnetic field so it is perpendicular to the plane of the page. The magnetic force on an electron has magnitude $F_B = evB$, and the acceleration of the electron has magnitude $a = v^2/r$. Newton's second law yields $evB = m_e v^2/r$, so the radius of the circle is given by $r = m_e v/eB$ in agreement with Eq. 28-16. The kinetic energy of the electron is $K = \frac{1}{2}m_e v^2$, so $v = \sqrt{2K/m_e}$. Thus,

$$r = \frac{m_e}{eB}\sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2m_e K}{e^2 B^2}} .$$

This must be less than d, so $\sqrt{\frac{2m_e K}{e^2 B^2}} \le d$, or $B \ge \sqrt{\frac{2m_e K}{e^2 d^2}}$.

(b) If the electrons are to travel as shown in Fig. 28-33, the magnetic field must be out of the page. Then the magnetic force is toward the center of the circular path, as it must be (in order to make the circular motion possible).