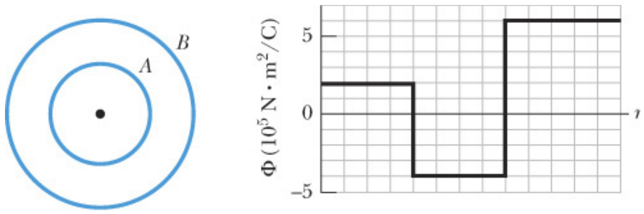
Gauss' Law Solution Set, Cha. 23 #12,13,20,24,27,47,49,50
due Fri, 2/2



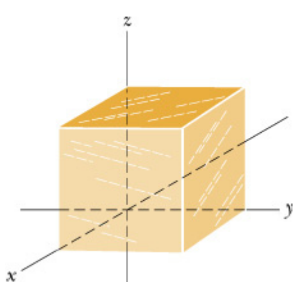
(a) (b)

12. Eq. 23-6 (Gauss' law) gives $\varepsilon_0\Phi = q_{\text{enclosed}}$.

(a) Thus, the value $\Phi = 2.0 \times 10^5$ (in SI units) for small r leads to $q_{\text{central}} = +1.77 \times 10^{-6}$ C or roughly 1.8 μC.

(b) The next value that Φ takes is -4.0×10^5 (in SI units), which implies $q_{\text{enc}} = -3.54\times10^{-6}\text{C}$. But we have already accounted for some of that charge in part (a), so the result for part (b) is $q_A = q_{\text{enc}} - q_{\text{central}} = -5.3 \times 10^{-6}$ C.

(c) Finally, the large r value for Φ is 6.0×10^5 (in SI units), which implies $q_{\text{total enc}} = 5.31\times10^{-6}\text{C}$. Considering what we have already found, then the result is $q_{\text{total enc}} - q_A - q_{\text{central}} = +8.9\mu C$.



13. (a) Let $A = (1.40\text{ m})^2$. Then

$$\Phi = \left(3.00y\,\hat{\text{j}}\right)\cdot\left(-A\,\hat{\text{j}}\right)\Big|_{y=0} + \left(3.00y\,\hat{\text{j}}\right)\cdot\left(\text{A}\,\hat{\text{j}}\right)\Big|_{y=1.40} = (3.00)(1.40)(1.40)^2 = 8.23\ \text{N}\cdot\text{m}^2/\text{C}.$$

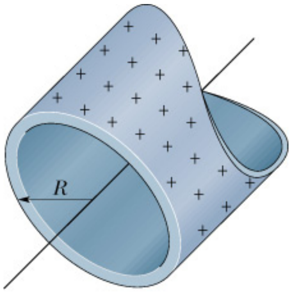
(b) The charge is given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$

(c) The electric field can be re-written as $\vec{E} = 3.00y\,\hat{\text{j}} + \vec{E}_0$, where $\vec{E}_0 = -4.00\hat{\text{i}} + 6.00\hat{\text{j}}$ is a constant field which does not contribute to the net flux through the cube. Thus Φ is still 8.23 N·m^2/C.

(d) The charge is again given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$



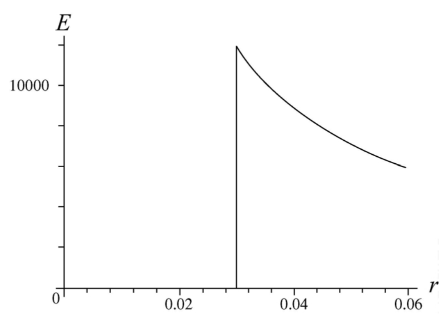
20. We imagine a cylindrical Gaussian surface A of radius r and unit length concentric with the metal tube. Then by symmetry $\oint_A \vec{E}\cdot d\vec{A} = 2\pi rE = \frac{q_{\text{enc}}}{\varepsilon_0}$.

(a) For $r < R$, $q_{\text{enc}} = 0$, so $E = 0$.

(b) For $r > R$, $q_{\text{enc}} = \lambda$, so $E(r) = \lambda / 2\pi r\varepsilon_0$. With $\lambda = 2.00\times10^{-8}$ C/m and $r = 2.00R = 0.0600$ m, we obtain

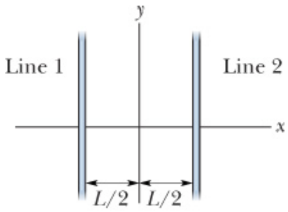
$$E = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.0600\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 5.99\times10^{3}\ \text{N/C}.$$

(c) The plot of E vs. r is shown below.



Here, the maximum value is

$$E_{\max} = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.030\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.2\times10^{4}\ \text{N/C}.$$



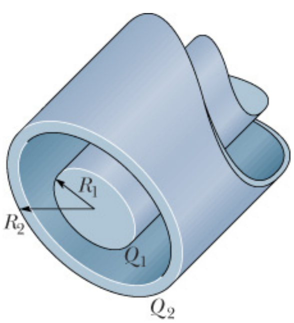
24. We reason that point P (the point on the x axis where the net electric field is zero) cannot be between the lines of charge (since their charges have opposite sign). We reason further that P is not to the left of "line 1" since its magnitude of charge (per unit length) exceeds that of "line 2"; thus, we look in the region to the right of "line 2" for P. Using Eq. 23-12, we have

$$E_{\text{net}} = E_1 + E_2 = \frac{\lambda_1}{2\pi\varepsilon_0 (x + L/2)} + \frac{\lambda_2}{2\pi\varepsilon_0 (x - L/2)} .$$

Setting this equal to zero and solving for x we find

$$x = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\,\frac{L}{2}$$

which, for the values given in the problem, yields $x = 8.0$ cm.



27. We assume the charge density of both the conducting cylinder and the shell are uniform, and we neglect fringing effect. Symmetry can be used to show that the electric field is radial, both between the cylinder and the shell and outside the shell. It is zero, of course, inside the cylinder and inside the shell.

(a) We take the Gaussian surface to be a cylinder of length L, coaxial with the given cylinders and of larger radius r than either of them. The flux through this surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. We may ignore any flux through the ends. Now, the charge enclosed by the Gaussian surface is $q_{\text{enc}} = Q1 + Q_2 = -Q_1 = -3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{\text{enc}}$, or

$$E = \frac{q_{\text{enc}}}{2\pi\varepsilon_0 Lr} = \frac{-3.40\times10^{-12}\ \text{C}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(11.0\ \text{m})(20.0\times1.30\times10^{-3}\text{m})} = -0.214\ \text{N/C},$$

or $|E| = 0.214$ N/C.

(b) The negative sign in E indicates that the field points inward.

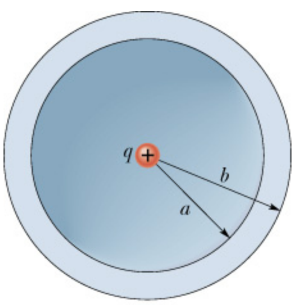
(c) Next, for r = 5.00 R_1, the charge enclosed by the Gaussian surface is $q_{\text{enc}} = Q_1 = 3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{\text{enc}}$, or

$$E = \frac{q_{\text{enc}}}{2\pi\varepsilon_0 Lr} = \frac{3.40\times10^{-12}\ \text{C}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(11.0\ \text{m})(5.00\times1.30\times10^{-3}\text{m})} = 0.855\ \text{N/C}.$$

(d) The positive sign indicates that the field points outward.

(e) we consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss' law). Since the central rod has charge Q_1, the inner surface of the shell must have charge $Q_{\text{in}} = -Q_1 = -3.40\times10^{-12}$ C.

(f) Since the shell is known to have total charge $Q_2 = -2.00Q_1$, it must have charge $Q_{\text{out}} = Q_2 - Q_{\text{in}} = -Q_1 = -3.40\times10^{-12}$ C on its outer surface.



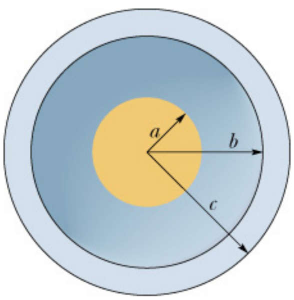
47. To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, select A so the field does not depend on the distance. We use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance r_g from the shell center. The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho\, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take dV to be the volume of a spherical shell with radius r and infinitesimal thickness dr: $dV = 4\pi r^2\, dr$. Thus,

$$q_s = 4\pi \int_a^{r_g} \rho r^2 dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 dr = 4\pi A \int_a^{r_g} r\, dr = 2\pi A \left(r_g^2 - a^2\right).$$

The total charge inside the Gaussian surface is $q+q_s=q+2\pi A\left(r_g^2-a^2\right)$. The electric field is radial, so the flux through the Gaussian surface is $\Phi=4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields $4\pi\varepsilon_0 E r_g^2=q+2\pi A\left(r_g^2-a^2\right)$. We solve for E:

$$E=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_g^2}+2\pi A-\frac{2\pi A a^2}{r_g^2}\right].$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q-2\pi A a^2=0$ or $A=q/2\pi a^2$. With $a=2.00\times10^{-2}$ m and $q=45.0\times10^{-15}$ C, we have $A=1.79\times10^{-11}\,\mathrm{C/m^2}$.



49. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\oint \vec{E}\cdot d\vec{A}=4\pi r^2 E$, where r is the radius of the Gaussian surface.

For $r<a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E=\left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \Rightarrow E=\frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(a) For $r=0$, the above equation implies $E=0$.

(b) For $r=a/2$, we have

$$E=\frac{q_1(a/2)}{4\pi\varepsilon_0 a^3}=\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\,\mathrm{C})}{2(2.00\times10^{-2}\,\mathrm{m})^2}=5.62\times10^{-2}\ \mathrm{N/C}.$$

(c) For $r=a$, we have

$$E=\frac{q_1}{4\pi\varepsilon_0 a^2}=\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\,\mathrm{C})}{(2.00\times10^{-2}\,\mathrm{m})^2}=0.112\ \mathrm{N/C}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is q_1, so Gauss' law leads to

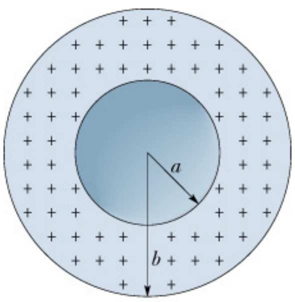
$$4\pi r^2 E = \frac{q_1}{\varepsilon_0} \Rightarrow E = \frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(d) For $r = 1.50a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\,\mathrm{C})}{(1.50\times2.00\times10^{-2}\,\mathrm{m})^2} = 0.0499\ \mathrm{N/C}.$$

(e) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$.

(f) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E = 0 \Rightarrow E = 0$. Thus, $E = 0$ at $r = 3.50a$.



50. The field is zero for $0 \le r \le a$ as a result of Eq. 23-16. Thus,

(a) $E = 0$ at $r = 0$,

(b) $E = 0$ at $r = a/2.00$, and

(c) $E = 0$ at $r = a$.

For $a \le r \le b$ the enclosed charge q_{enc} (for $a \le r \le b$) is related to the volume by

$$q_{enc} = \rho\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right).$$

Therefore, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2} = \frac{\rho}{4\pi\varepsilon_0 r^2}\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right) = \frac{\rho}{3\varepsilon_0}\frac{r^3 - a^3}{r^2}$$

for $a \le r \le b$.

(d) For $r = 1.50a$, we have

$$E = \frac{\rho}{3\varepsilon_0}\frac{(1.50a)^3 - a^3}{(1.50a)^2} = \frac{\rho a}{3\varepsilon_0}\frac{2.375}{2.25} = \frac{(1.84\times10^{-9})(0.100)}{3(8.85\times10^{-12})}\frac{2.375}{2.25} = 7.32 \text{ N/C}.$$

(e) For $r = b = 2.00a$, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(2.00a)^2} = \frac{\rho a}{3\varepsilon_0}\frac{7}{4} = \frac{(1.84\times10^{-9})(0.100)}{3(8.85\times10^{-12})}\frac{7}{4} = 12.1 \text{ N/C}.$$

(f) For $r \geq b$ we have $E = q_{\text{total}} / 4\pi\varepsilon_0 r^2$ or

$$E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2}.$$

Thus, for $r = 3.00b = 6.00a$, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(6.00a)^2} = \frac{\rho a}{3\varepsilon_0}\frac{7}{36} = \frac{(1.84\times10^{-9})(0.100)}{3(8.85\times10^{-12})}\frac{7}{4} = 1.35 \text{ N/C}.$$

notes