AP Solutions, Circuits: Cha 27 part 1

6,19,23,25,30 due Fri. 3/2

Mr. Shapiro

6. The current in the circuit is

$$i = (150\text{ V} - 50\text{ V})/(3.0\ \Omega + 2.0\ \Omega) = 20\text{ A}.$$

So from $V_Q + 150\text{ V} - (2.0\ \Omega)i = V_P$, we get $V_Q = 100\text{ V} + (2.0\ \Omega)(20\text{ A}) - 150\text{ V} = -10\text{ V}$.

19. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward.

(a) When the loop rule is applied to the lower loop, the result is

$$\varepsilon_2 - i_1 R_1 = 0$$

The equation yields

$$i_1 = \frac{\varepsilon_2}{R_1} = \frac{5.0\text{ V}}{100\Omega} = 0.050\text{ A}.$$

(b) When it is applied to the upper loop, the result is

$$\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - i_2 R_2 = 0\ .$$

The equation yields

$$i_2 = \frac{\varepsilon_1 - \varepsilon_2 - \varepsilon_3}{R_2} = \frac{6.0\text{ V} - 5.0\text{ V} - 4.0\text{ V}}{50\Omega} = -0.060\text{ A}\,,$$

or $|i_2| = 0.060$ A. The negative sign indicates that the current in R_2 is actually downward.

(c) If V_b is the potential at point b, then the potential at point a is $V_a = V_b + \varepsilon_3 + \varepsilon_2$, so $V_a - V_b = \varepsilon_3 + \varepsilon_2 = 4.0\text{ V} + 5.0\text{ V} = 9.0\text{ V}$.

23. First, we note V_4, that the voltage across R_4 is equal to the sum of the voltages across R_5 and R_6:

$$V_4 = i_6(R_5 + R_6) = (1.40\text{ A})(8.00\ \Omega + 4.00\ \Omega) = 16.8\text{ V}.$$

The current through R_4 is then equal to $i_4 = V_4/R_4 = 16.8\text{ V}/(16.0\ \Omega) = 1.05\text{ A}$.

By the junction rule, the current in R_2 is $i_2 = i_4 + i_6 = 1.05\text{ A} + 1.40\text{ A} = 2.45\text{ A}$, so its voltage is $V_2 = (2.00\ \Omega)(2.45\text{ A}) = 4.90\text{ V}$.

The loop rule tells us the voltage across R_3 is $V_3 = V_2 + V_4 = 21.7$ V (implying that the current through it is $i_3 = V_3/(2.00\ \Omega) = 10.85$ A).

The junction rule now gives the current in R_1 as $i_1 = i_2 + i_3 = 2.45\text{ A} + 10.85\text{ A} = 13.3$ A, implying that the voltage across it is $V_1 = (13.3\text{ A})(2.00\ \Omega) = 26.6$ V. Therefore, by the loop rule,

$$\varepsilon = V_1 + V_3 = 26.6\text{ V} + 21.7\text{ V} = 48.3\text{ V}.$$

25. The voltage difference across R_3 is $V_3 = \varepsilon R'/(R' + 2.00\ \Omega)$, where

$$R' = (5.00\ \Omega R)/(5.00\ \Omega + R_3).$$

Thus,

$$P_3 = \frac{V_3^2}{R_3} = \frac{1}{R_3}\left(\frac{\varepsilon R'}{R' + 2.00\ \Omega}\right)^2 = \frac{1}{R_3}\left(\frac{\varepsilon}{1 + 2.00\ \Omega/R'}\right)^2 = \frac{\varepsilon^2}{R_3}\left[1 + \frac{(2.00\ \Omega)(5.00\ \Omega + R)}{(5.00\ \Omega)R_3}\right]^{-2}$$
$$\equiv \frac{\varepsilon^2}{f(R_3)}$$

where we use the equivalence symbol $\equiv$ to define the expression $f(R_3)$. To maximize P_3 we need to minimize the expression $f(R_3)$. We set

$$\frac{df(R_3)}{dR_3} = -\frac{4.00\ \Omega^2}{R_3^{\ 2}} + \frac{49}{25} = 0$$

to obtain $R_3 = \sqrt{(4.00\ \Omega^2)(25)/49} = 1.43\ \Omega.$

30. (a) R_2, R_3 and R_4 are in parallel. By finding a common denominator and simplifying, the equation $1/R = 1/R_2 + 1/R_3 + 1/R_4$ gives an equivalent resistance of

$$R = \frac{R_2R_3R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{(50.0\Omega)(50.0\Omega)(75.0\Omega)}{(50.0\Omega)(50.0\Omega) + (50.0\Omega)(75.0\Omega) + (50.0\Omega)(75.0\Omega)}$$
$$= 18.8\Omega.$$

Thus, considering the series contribution of resistor R_1, the equivalent resistance for the network is $R_{eq} = R_1 + R = 100\ \Omega + 18.8\ \Omega = 118.8\ \Omega \approx 119\ \Omega$.

(b) $i_1 = \varepsilon/R_{eq} = 6.0\text{ V}/(118.8\ \Omega) = 5.05 \times 10^{-2}$ A.

(c) $i_2 = (\varepsilon - V_1)/R_2 = (\varepsilon - i_1R_1)/R_2 = [6.0\text{V} - (5.05 \times 10^{-2}\text{ A})(100\Omega)]/50\ \Omega = 1.90 \times 10^{-2}$ A.

(d) $i_3 = (\varepsilon - V_1)/R_3 = i_2R_2/R_3 = (1.90 \times 10^{-2}\text{ A})(50.0\ \Omega/50.0\ \Omega) = 1.90 \times 10^{-2}$ A.

(e) $i_4 = i_1 - i_2 - i_3 = 5.05 \times 10^{-2}\text{ A} - 2(1.90 \times 10^{-2}\text{ A}) = 1.25 \times 10^{-2}$ A.