AP Physics - Mr. Shapiro-2018

Cha. 27 problem set part 2 #44-51,53,54 due Tues. 3/9

44. (a) We use $q = q_0 e^{-t/\tau}$, or $t = \tau \ln (q_0/q)$, where $\tau = RC$ is the capacitive time constant. Thus, $t_{1/3} = \tau \ln[q_0/(2q_0/3)] = \tau \ln(3/2) = 0.41\tau$, or $t_{1/3}/\tau = 0.41$.

(b) $t_{2/3} = \tau \ln[q_0/(q_0/3)] = \tau \ln 3 = 1.1\tau$, or $t_{2/3}/\tau = 1.1$.

45. During charging, the charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where C is the capacitance, ε is applied emf, and $\tau = RC$ is the capacitive time constant. The equilibrium charge is $q_{eq} = C\varepsilon$. We require $q = 0.99q_{eq} = 0.99C\varepsilon$, so

$$0.99 = 1 - e^{-t/\tau}.$$

Thus, $e^{-t/\tau} = 0.01$. Taking the natural logarithm of both sides, we obtain $t/\tau = -\ln 0.01 = 4.61$ or $t = 4.61\tau$.

46. (a) $\tau = RC = (1.40 \times 10^6\ \Omega)(1.80 \times 10^{-6}\ \text{F}) = 2.52$ s.

(b) $q_o = \varepsilon C = (12.0\ \text{V})(1.80\ \mu\text{F}) = 21.6\ \mu\text{C}$.

(c) The time t satisfies $q = q_0(1 - e^{-t/RC})$, or

$$t = RC\ln\left(\frac{q_0}{q_0 - q}\right) = (2.52\,\text{s})\ln\left(\frac{21.6\,\mu\text{C}}{21.6\,\mu\text{C} - 16.0\,\mu\text{C}}\right) = 3.40\,\text{s}.$$

47. (a) The voltage difference V across the capacitor is $V(t) = \varepsilon(1 - e^{-t/RC})$. At $t = 1.30\ \mu$s we have $V(t) = 5.00$ V, so $5.00\ \text{V} = (12.0\ \text{V})(1 - e^{-1.30\ \mu\text{s}/RC})$, which gives

$$\tau = (1.30\ \mu\text{s})/\ln(12/7) = 2.41\ \mu\text{s}.$$

(b) $C = \tau/R = 2.41\ \mu\text{s}/15.0\ \text{k}\Omega = 161$ pF.

48. Here we denote the battery emf as V. Then the requirement stated in the problem that the resistor voltage be equal to the capacitor voltage becomes $iR = V_{cap}$, or

$$V\mathrm{e}^{-t/RC} = V(1 - \mathrm{e}^{-t/RC})$$

where Eqs. 27-34 and 27-35 have been used. This leads to $t = RC\ln 2$, or $t = 0.208$ ms.

49. (a) The potential difference V across the plates of a capacitor is related to the charge q on the positive plate by $V = q/C$, where C is capacitance. Since the charge on a discharging capacitor is given by $q = q_0 e^{-t/\tau}$, this means $V = V_0 e^{-t/\tau}$ where V_0 is the initial potential difference. We solve for the time constant τ by dividing by V_0 and taking the natural logarithm:

$$\tau = -\frac{t}{\ln(V/V_0)} = -\frac{10.0\,\mathrm{s}}{\ln\left[(1.00\,\mathrm{V})/(100\,\mathrm{V})\right]} = 2.17\,\mathrm{s}.$$

(b) At $t = 17.0$ s, $t/\tau = (17.0\text{ s})/(2.17\text{ s}) = 7.83$, so

$$V = V_0 e^{-t/\tau} = (100\,\mathrm{V})e^{-7.83} = 3.96\times10^{-2}\ \mathrm{V}\,.$$

50. The potential difference across the capacitor varies as a function of time t as $V(t) = V_0 e^{-t/RC}$. Using $V = V_0/4$ at $t = 2.0$ s, we find

$$R = \frac{t}{C\ln(V_0/V)} = \frac{2.0\,\mathrm{s}}{(2.0\times10^{-6}\,\mathrm{F})\ln 4} = 7.2\times10^{5}\,\Omega.$$

51. (a) The initial energy stored in a capacitor is given by $U_C = q_0^2/2C$, where C is the capacitance and q_0 is the initial charge on one plate. Thus

$$q_0 = \sqrt{2CU_C} = \sqrt{2\left(1.0\times10^{-6}\ \text{F}\right)\left(0.50\ \text{J}\right)} = 1.0\times10^{-3}\ \text{C}\ .$$

(b) The charge as a function of time is given by $q = q_0 e^{-t/\tau}$, where τ is the capacitive time constant. The current is the derivative of the charge

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau}e^{-t/\tau}\ ,$$

and the initial current is $i_0 = q_0/\tau$. The time constant is

$$\tau = RC = \left(1.0\times10^{-6}\ \text{F}\right)\left(1.0\times10^{6}\ \Omega\right) = 1.0\ \text{s}.$$

Thus $i_0 = \left(1.0\times10^{-3}\ \text{C}\right)\big/\left(1.0\ \text{s}\right) = 1.0\times10^{-3}\ \text{A}$.

(c) We substitute $q = q_0\, e^{-t/\tau}$ into $V_C = q/C$ to obtain

$$V_C = \frac{q_0}{C}e^{-t/\tau} = \left(\frac{1.0\times10^{-3}\ \text{C}}{1.0\times10^{-6}\ \text{F}}\right)e^{-t/1.0\ \text{s}} = \left(1.0\times10^{3}\ \text{V}\right)e^{-1.0t}\ ,$$

where t is measured in seconds.

(d) We substitute $i = (q_0/\tau)e^{-t/\tau}$ into $V_R = iR$ to obtain

$$V_R = \frac{q_0 R}{\tau}e^{-t/\tau} = \frac{\left(1.0\times10^{-3}\ \text{C}\right)\left(1.0\times10^{6}\ \Omega\right)}{1.0\ \text{s}}e^{-t/1.0\ \text{s}} = \left(1.0\times10^{3}\ \text{V}\right)e^{-1.0t}\ ,$$

where t is measured in seconds.

(e) We substitute $i = (q_0/\tau)e^{-t/\tau}$ into $P = i^2R$ to obtain

$$P = \frac{q_0^2 R}{\tau^2}e^{-2t/\tau} = \frac{\left(1.0\times10^{-3}\ \text{C}\right)^2\left(1.0\times10^{6}\ \Omega\right)}{\left(1.0\ \text{s}\right)^2}e^{-2t/1.0\ \text{s}} = \left(1.0\ \text{W}\right)e^{-2.0t}\ ,$$

where t is again measured in seconds.

53. At $t = 0$ the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0 ,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0 .$$

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R.

(a) Solving the three simultaneous equations, we find

$$i_1 = \frac{2\varepsilon}{3R} = \frac{2\left(1.2\times10^3\text{ V}\right)}{3\left(0.73\times10^6\,\Omega\right)} = 1.1\times10^{-3}\text{ A}$$

(b) $i_2 = \dfrac{\varepsilon}{3R} = \dfrac{1.2\times10^3\text{ V}}{3\left(0.73\times10^6\,\Omega\right)} = 5.5\times10^{-4}\text{ A}.$

(c) $i_3 = i_2 = 5.5\times10^{-4}\text{ A}.$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields

$$\varepsilon - i_1 R_1 - i_1 R_2 = 0 .$$

(d) The solution is

$$i_1 = \frac{\varepsilon}{2R} = \frac{1.2\times10^3\text{ V}}{2\left(0.73\times10^6\,\Omega\right)} = 8.2\times10^{-4}\text{ A}.$$

(e) $i_2 = i_1 = 8.2\times10^{-4}\text{ A}.$

(f) As stated before, the current in the capacitor branch is $i_3 = 0$.

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54. In the steady state situation, the capacitor voltage will equal the voltage across R_2 = 15 kΩ:

$$V_0 = R_2 \frac{\varepsilon}{R_1 + R_2} = (15.0\,\text{k}\Omega)\left(\frac{20.0\,\text{V}}{10.0\,\text{k}\Omega + 15.0\,\text{k}\Omega}\right) = 12.0\,\text{V}.$$

Now, multiplying Eq. 27-39 by the capacitance leads to $V = V_0 e^{-t/RC}$ describing the voltage across the capacitor (and across R_2 = 15.0 kΩ) after the switch is opened (at t = 0). Thus, with t = 0.00400 s, we obtain

$$V = (12) e^{-0.004/(15000)(0.4\times10^{-6})} = 6.16\,\text{V}.$$

Therefore, using Ohm's law, the current through R_2 is 6.16/15000 = 4.11×10^{-4} A.